6/12/2015
سكشن $\boxed{8}$ رياضة

AMR M.R

9

Series of O.D.E.

حل المعادلة التفاضلية باستخدام المتسلسلة.

$$1.\ y'' + P(x)y' + Q(x)y = F(x)$$

① O.P

$P(0) \neq \infty$

$Q(0) \neq \infty$

$$\boxed{y = \sum_{n=0}^{\infty} a_n x^n}$$

Solve:

$\boxed{\text{ex}}$ $y'' + y = 0 \quad , \quad x_0 = 0$

$P(x) = 0 \neq \infty \quad , \quad Q(x) = 1 \neq \infty \quad \}$ O.P

$$\therefore y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$$

بالتعويض من $y\,(y'')$

$$\sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0$$

Co.of $x^0 \longrightarrow 2a_2 + a_0 = 0$

$$\boxed{a_2 = \frac{-a_0}{2}} \longrightarrow ①$$

Co.of $x^1 \longrightarrow 3*2*a_3 + a_1 = 0$

$$\boxed{a_3 = \frac{-a_1}{6}} \longrightarrow ②$$

Co.of x^n

$(n+2)(n+1)a_{n+2} + a_n = 0$

$$③ \rightarrow \boxed{a_{n+2} = \frac{-a_n}{(n+2)(n+1)}}$$

بالتعويض من ①

at $n = 2$

$$a_4 = \frac{-a_2}{4*3} = \frac{+a_0}{24}$$

$\therefore$ الحل

$$y = a_0 + a_1 x - \frac{a_0}{2}x^2 - \frac{a_1}{6}x^3 + \frac{a_0}{24}x^4 + \dots$$

6

الأذكار

① $t = x - x_0 \longrightarrow x_0 \neq 0$

$\boxed{ex}$ $y'' + xy' + y = 0 \quad , \quad x_0 = 1$

$$t = x - 1$$

$$y'' + (t-1)y' + y = 0 \quad , \quad t_0 = 0$$

$$y = \sum_{t=0}^{\infty} a_n t^n$$

---

② شرط في المسألة :-

نحل عادي وفي الأخر الآمر نجيب الثوابت a_1 ، a_0

---

③ كثيرة الحدود :-

نساوي معاملات الطرف اليمين و الحد العام .

---

④ دوال مثلثية أو لوغاريتمية أو آسية :-

نفكها .

$\boxed{ex}$ $y'' + y' - xy = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 1$

$P(x) = 1 \qquad Q(x) = -x \Big\} \ \underline{O.P}$

$$\therefore y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

(c)

Co. of x^0

$$2a_2 + 1a_1 = 0$$

$$\boxed{a_2 = \frac{-a_1}{2}} \longrightarrow (1)$$

Co. of (x^1)

$$6a_3 + 2a_2 - a_0 = 0$$

$$\boxed{a_3 = \frac{-2a_2 + a_0}{6}}$$

بالتعويض من [1]

$$\boxed{a_3 = \frac{a_1 + a_0}{6}}$$

Co. of (x^n)

$$(n+2)(n+1)\,a_{n+2} + (n+1)\,a_{n+1} - a_{n-1} = 0$$

$$a_{n+2} = \frac{-(n+1)\,a_{n+1} + a_{n-1}}{(n+2)(n+1)}$$

at n=2

$$a_4 = \frac{-3a_3 + a_1}{4 * 3} = \frac{-\frac{a_0 + a_1}{2} + a_1}{12}$$

$$a_4 = \frac{-a_0}{24} + \frac{a_1}{24}$$

∴ the solution

$$\therefore\ y = a_0 + a_1 x - \frac{a_1}{2}x^2 - \frac{a_0 a_1}{2}x^3 + \left(\frac{-a_0}{24} + \frac{a_1}{24}\right)x^4 + \dots$$

at $x = 0 \longrightarrow y = 2$

at $x = 0 \longrightarrow y' = 1$

$\therefore\ 2 = a_0$

$y' = a_1 - a_1 x + \dots$

ثم نعوض a_1 , a_0 في المعادلة الرئيسية

d

ex $(x^2+4)y'' + xy = x+2$

$P(x) = 0 \quad , \quad Q(x) = \frac{x}{x^2+4} = 0 \neq \infty \Big\} \text{ O.P}$

$y = \sum_{n=0}^{\infty} a_n x^n$

$y'' = \sum n(n-1) a_n x^{n-2}$

$\sum n(n-1) a_n x^n + 4 \sum n(n-1) a_n x^{n-2} + \sum a_n x^{n+1} = x+2$

Co. of x^0 = $(4*2) a_2 = 2$

$\boxed{a_2 = \frac{1}{4}} \rightarrow 1$

Co. of x^1

$24 a_3 + a_0 = 1$

$\boxed{a_3 = \frac{1-a_0}{24}}$

Co. of x^n

$n(n-1) a_n + 4(n+2)(n+1) a_{n+2} + a_{n-1} = 0$

أكبر واحدة

$a_{n+2} = \frac{-n(n-1) a_n - a_{n-1}}{4(n+2)(n+1)}$

at $n=2$ $\quad a_4 = \frac{-2a_2 - a_1}{4*4*3} = \frac{-a_1 - 1/2}{48}$

$\therefore y = a_0 + a_1 x + \frac{1}{4} x^2 + \frac{1-a_0}{24} x^3 + \frac{-a_1 - 1/2}{48} x^4 + \cdots$

$\boxed{e}$

$\boxed{Ex}$

$$x y'' + \underline{\sin x}\, y = 0$$

$P(x) = 0$ $\qquad Q(x) = \frac{\sin x}{x} = \frac{0}{0} =$

$\lim \frac{\sin x}{x} = 1$ $\Big\}$ o.p

مفكوك $\sin x$

$$x y'' + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right) y = 0$$

$$y = \sum a_n x^n$$

$$y'' = \sum n(n-1) a_n x^{n-2}$$

$$\sum n(n-1) a_n x^{n-1} + \sum a_n x^{n+1} - \frac{1}{3!} \sum a_n x^{n+3} + \frac{1}{5!} \sum a_n x^{n+5}$$

$$+ \cdots = 0$$

<u>Co. of x^0</u> $\rightarrow$

<u>لا يوجد</u>

<u>Co. of x^1</u> $\rightarrow$ $2 * a_2 + a_0 = 0$

$$\boxed{a_2 = \frac{-a_0}{2}}$$

Co. of x^n $\longrightarrow$ $(n+1) n\, a_{n+1} + a_{n-1} - \frac{1}{3!} a_{n-3} + \frac{1}{5!} a_{n-5}$

$$+ \cdots = 0$$

$$\therefore a_{n+1} = \frac{-a_{n-1} + \frac{1}{3!} a_{n-3} - \frac{1}{5!} a_{n-5} + \cdots}{n(n+1)}$$

<u>at $n=2$</u> $\qquad a_3 = \frac{-a_1}{6}$

$$\therefore y = a_0 + a_1 x + \frac{-a_0}{2} x^2 - \frac{a_1}{6} x^3 + \cdots$$